

Regular Expressions

A **Regular Expression** can be recursively defined as follows –

- ϵ is a Regular Expression indicates the language containing an empty string. ($L(\epsilon) = \{\epsilon\}$)
- ϕ is a Regular Expression denoting an empty language. ($L(\phi) = \{\}$)
- x is a Regular Expression where $L = \{x\}$
- If X is a Regular Expression denoting the language $L(X)$ and Y is a Regular Expression denoting the language $L(Y)$, then
 - $X + Y$ is a Regular Expression corresponding to the language $L(X) \cup L(Y)$ where $L(X+Y) = L(X) \cup L(Y)$.
 - $X \cdot Y$ is a Regular Expression corresponding to the language $L(X) \cdot L(Y)$ where $L(X \cdot Y) = L(X) \cdot L(Y)$
 - R^* is a Regular Expression corresponding to the language $L(R^*)$ where $L(R^*) = (L(R))^*$
- If we apply any of the rules several times from 1 to 5, they are Regular Expressions.

Some RE Examples

Regular Expressions	Regular Set
$(0 + 10^*)$	$L = \{ 0, 1, 10, 100, 1000, 10000, \dots \}$
(0^*10^*)	$L = \{1, 01, 10, 010, 0010, \dots\}$
$(0 + \epsilon)(1 + \epsilon)$	$L = \{\epsilon, 0, 1, 01\}$
$(a+b)^*$	Set of strings of a's and b's of any length including the null string. So $L = \{ \epsilon, a, b, aa, ab, bb, ba, aaa, \dots \}$
$(a+b)^*abb$	Set of strings of a's and b's ending with the string abb. So $L = \{abb, aabb, babb, aaabb, ababb, \dots\}$
$(11)^*$	Set consisting of even number of 1's including empty string, So $L = \{\epsilon, 11, 1111, 111111, \dots\}$
$(aa)^*(bb)^*b$	Set of strings consisting of even number of a's followed by odd number of b's, so $L = \{b, aab, aabbb, aabbbb, aaaab, aaaabbb, \dots\}$
$(aa + ab + ba + bb)^*$	String of a's and b's of even length can be obtained by concatenating any combination of the strings aa, ab, ba and bb including null, so $L = \{aa, ab, ba, bb, aaab, aaba, \dots\}$